CHAPTER 4

Polynomials (Remainder and Factor Theorem)

4.1 Introduction

$2x^2 + 3x + 1$, $3x^2 - 2x^2 + x - 2$, $5x^4 + 4x^3 - 5x^2 + x + 3$.....

These are polynomials in x of different degree. The exponents in the powers of x are non-negative integers and the coefficients of various powers of x are integers. Such polynomials are called Rational Integral functions of x.

Given a rational integral function f(x), can we find the remainder when f(x) is divided by (x – a) without actual division? The answer is affirmative and is provided by the following theorem called "remainder theorem". Before taking up the theorem we shall observe the following examples.

(i) When $x^2 + x - 5$ is divided by $(x - 2)$ the quotient is $(x + 3)$ and remainder is 1.
$\therefore\ x^2 + x - 5 = (x + 3)(x - 2) + 1$

(ii) When $x^3 + x^2 - 4x - 2$ is divided by $(x + 1)$ the quotient is $(x^2 - 4)$ and remainder is 2.
$\therefore\ x^3 + x^2 - 4x - 2 = (x + 1)(x^2 - 4) + 2$

(iii) When $x^3 + 2x^2 - 9x - 20$ is divided by $(x + 3)$ the quotient is $(x^2 - x - 6)$ and the remainder is –2.
$\therefore\ x^3 + 2x^2 - 9x - 20 = (x + 3)(x^2 - x - 6) - 2$

These examples illustrate that if a polynomial is divided by a first degree binomial, the quotient is a polynomial whose degree is one less than the degree of the dividend and the remainder is constant.

(i) Substituting zero of $(x - 2)$ in $x^2 + x - 5$
$f(x) = x^2 + x - 5 = f(2) = (2)^2 + (2) - 5 = 1$

(ii) Substituting zero of $(x + 1)$ in $x^3 + x^2 - 4x - 2$
$f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 2 = 2$

(iii) Substituting zero of $(x + 3)$ in $x^3 + 2x^2 - 9x - 20$
$f(-3) = (-3)^3 + 2(-3)^2 - 9(-3) - 20 = -2$

4.2 Remainder theorem

If a rational integral function of x, say f(x) is divided by (x–a) then the remainder is f(a).

Proof

Let Q(x) be the quotient and R the remainder when f(x) is divided by (x–a).

Then we have f(x) = Q(x)(x–a) + R, where the degree of Q is one less than that of f(x) and R is a constant.

Since the equation is true for all real value of x putting x = a in the above, we get f(a)=Q(x) (a–a) + R

i.e., $R = f(a)$ $\therefore\ f(x) = (x - a)\,Q(x) + f(a)$

4.3 Factor theorem

If f(x) is a rational integral function of x and $f(a) = 0$ then $(x - a)$ is a factor of f(x).

Proof

We have, by Remainder theorem,

$f(x) = (x - a)\,Q(x)$. This shows that $(x - a)$ is a factor of f(x).

- If f(x) denotes a polynomial function of degree ≥ 1 and if $r \in R$ then there is a polynomial function denoted by p(x) such that for all $r \in R$.

  $f(x) = (x - r)\,p(x) + f(r)$

  If $f(r) = 0$ then $f(x) = (x - r)\,p(x)$

  $(x - r)$ is a factor of f(x) iff $f(r) = 0$

  This is called the factor theorem.

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- If f and g are two polynomials then f is a factor of g iff there is some polynomial Q such that the equation g = f.Q is an identity.
- $x^n + y^n$ is always divisible by (x + y) where n∈ odd natural numbers.
- $x^n - y^n$ is always divisible by (x – y) where n∈ odd natural numbers.
- $x^n - y^n$ is always divisible by (x + y) and (x – y) where n∈ even natural numbers.
- Some times simple inspection about the coefficients of f(x) yields some of the factors of the given expression.
  Let $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + a_3 x^{n-3} + a_4 x^{n-4} + a_5 x^{n-5} + \ldots\ldots\ldots + a_n = 0$
  (a) (x – 1) is a factor of f(x) iff
  $a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + \ldots\ldots + a_n = 0$
  For (x – 1) is a factor of f(x) ⇔ f(1) = 0.
  i.e., $a_0\ 1^n + a_1\ 1^{n-1} + a_2\ 1^{n-2} + \ldots\ldots + a_n = 0$
  i.e., $a_0 + a_1 + a_2 + \ldots\ldots + a_n = 0$
  i.e., (x – 1) is a factor of f(x) iff the sum of the coefficients of powers of x in f(x) is zero.
  (b) (x + 1) is a factor of f(x) if the sum of the coefficients of even powers of x is equal to the sum of the coefficients of odd powers of x.
  For example, if (x + 1) is a factor of f(x) then f(–1) = 0. This gives
  $a_0 (-1)^n + a_1(-1)^{n-1} + a_2 (-1)^{n-2} + a_3(-1)^{n-3} + a_4 (-1)^{n-4} + a_5(-1)^{n-5} \ldots\ldots + a_n = 0$
  i.e., $a_0 + a_2 + a_4 + \ldots\ldots = a_1 + a_3 + a_5 + \ldots\ldots\ldots$
  In both the cases i.e., when 'n' is odd and 'n' is even the converse is also true.
  This shows that the sum of coefficients of even powers of 'x' is equal to the sum of the coefficients of odd powers of x iff (x + 1) is a factor of f(x).

Solving cubic equations

Having solved linear and quadratic equations, we shall now consider cubic equations of the form $ax^3 + bx^2 + cx + d = 0$, where a, b, c, d are constants.

- Steps in solving cubic equations
  Step 1 : Let f(x) be the expression.
  Step 2 : Find a factor (x – α) by trial and error method, where α is constant.
  Step 3 : The other two factor can be found by:
  (i) Comparing coefficients
  (ii) Long division
  (iii) Synthetic method
  Step 4 : Write down f(x) = (x – α)($px^2 + qx + r$) = 0 and obtain the answer.

NUMERICAL CHALLENGE 4.1

Find the remainder when $4x^2 - 8x + 3$ is divided by
(i) 2x – 1 (ii) 2x + 1 (iii) 2x – 3 (iv) 2x + 3

Solution

(i) Remainder = $f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^2 - 8\left(\frac{1}{2}\right) + 3$

$= 4 \times \frac{1}{4} - 8 \times \frac{1}{2} + 3 = 0$

(ii) Remainder = $f\left(-\frac{1}{2}\right) = 4\left(\frac{-1}{2}\right)^2 - 8\left(\frac{-1}{2}\right) + 3$

$= 4 \times \frac{1}{4} + 8 \times \frac{1}{2} + 3 = 8$

(iii) Remainder = $f\left(\frac{3}{2}\right) = 4\left(\frac{3}{2}\right)^2 - 8\left(\frac{3}{2}\right) + 3 = 9 - 12 + 3 = 0$

(iv) Remainder = $f\left(\frac{-3}{2}\right) = 4\left(\frac{-3}{2}\right)^2 - 8\left(\frac{-3}{2}\right) + 3 = 9 + 12 + 3 = 24$

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NUMERICAL CHALLENGE 4.2

If $x^3 + 2x^2 + ax + b$ has factors $x + 1$ and $x - 1$, find a and b.

Solution

If $(x - 1)$ is a factor of $f(x)$ we should have $f(1) = 0$

This gives $1^3 + 2.1^2 + a.1 + b = 0$

$\therefore$ $a + b = -3$ .....(1)

$(x + 1)$ is a factor of $f(x)$ we should have $f(-1) = 0$

This gives $(-1)^3 + 2(-1)^2 + a(-1) + b = 0$

$\therefore -1 + 2 - a + b = 0$

$-a + b = -1$

i.e., $a - b = 1$ .....(2)

Adding (2) and (1), we get

$2a = -2 \therefore a = -1$, Using $a = -1$ in either (1) or (2), we get $b = -2$

NUMERICAL CHALLENGE 4.3

Factorise $x^4 + 5x^3 + 5x^2 - 5x - 6$

Solution

$f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$

Consider $f(1) = 1^4 + 5.1^3 + 5.1^2 - 5.1 - 6 = 0$

and $f(-1) = (-1)^4 + 5(-1)^3 + 5(-1)^2 - 5(-1) - 6 = 0$

$\therefore$ $(x - 1)$, $(x + 1)$ are two factors of $f(x)$

$\therefore$ $(x - 1)(x + 1) = (x^2 - 1)$

Dividing $f(x)$ by $(x^2 - 1) = (x - 1)(x + 1)$, we get $x^2 + 5x + 6$ as quotient.

$\therefore$ $f(x) = (x - 1)(x + 1)(x^2 + 5x + 6)$

$\therefore$ $f(x) = (x - 1)(x + 1)(x + 3)(x + 2)$

NUMERICAL CHALLENGE 4.4

If $ax^2 + bx + c$ is exactly divisible by $(x - 1)$, $(x - 2)$ and leaves remainder 6 when divided by $(x + 1)$, find a, b and c.

Solution

If $f(x) = ax^2 + bx + c$, then by the hypothesis

$f(1) = a + b + c = 0$ .....(1)

$f(2) = 4a + 2b + c = 0$ .....(2)

$f(-1) = a - b + c = 6$ .....(3)

Subtracting equation (3) from equation (1)

$\Rightarrow 2b = -6$, So, $b = -3$

Using $b = -3$ in (1) and (2), we get

$a + c = 3$ .....(4)

$4a + c = 6$ .....(5)

Solving (4) and (5), we get $a = 1$ and $c = 2$

$\therefore$ $a = 1$, $b = -3$ and $c = 2$

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NUMERICAL CHALLENGE 4.5

Find the remainder when x^{91} is divided by $x^2 - 1$.

Solution

Let $hx + r$ be the remainder and $q(x)$ be the quotient when x^{91} is divided by $x^2 - 1$.

Then $x^9 = q(x)(x^2 - 1) + hx + r$ ..... (1)

Putting $x = 1$ in equation (1)

$1 = q(x) \times 0 + h \times 1 + r$

$\Rightarrow h + r = 1$ ..... (2)

Putting $x = -1$ in equation (1)

$-1 = q(x) \times 0 + h \times -1 + r$

$-h + r = -1$ ..... (3)

Adding equation (2) & (3) we get

$2r = 0 \Rightarrow r = 0$

Putting $r = 0$ in equation (1) we get $h = 1$.

Thus remainder $= hx + r = 1 \times x + 0 = x$.

NUMERICAL CHALLENGE 4.6

Find the remainder and the quotient when $4x^3 - 3x + 9$ is divided by $2x - 3$.

Solution

Let $f(x) = 4x^3 - 3x + 9$. When divided by $2x - 3$ the remainder is $f\left(\frac{3}{2}\right)$

$$f\left(\frac{3}{2}\right) = 4\left(\frac{3}{2}\right)^3 - 3\left(\frac{3}{2}\right) + 9 = \frac{4 \times 27}{8} - \frac{9}{2} + 9 = 18$$

Remainder $= f\left(\frac{3}{2}\right) = 18$

Now on dividing $f(x)$ by the divisor $2x - 3$ the quotient is $2x^2 + 3x + 3$

Therefore, when the given polynomial $f(x)$ is divided by $2x - 3$, the quotient will be $2x^2 + 3x + 3$ and remainder will be 18

$$\begin{array}{r|l} & 2x^2 + 3x + 3 \\ 2x - 3 & \overline{)\ 4x^3 - 3x + 9\ (} \\ & \underline{-4x^3} \\ & 6x^2 - 3x + 9 \\ & \underline{-6x^2 \mp 9x} \\ & 6x + 9 \\ & \underline{-6x \mp 9} \\ & 18 \end{array}$$

NUMERICAL CHALLENGE 4.7

The remainder of the polynomial $5 + bx - 2x^2 + ax^3$ when divided by $x - 2$ is twice the remainder when it is divided by $x + 1$. Show that $10a + 4b = 9$.

Solution

Let $f(x) = 5 + bx - 2x^2 + ax^3$

$f(2) = 5 + b(2) - 2(2)^2 + a(2)^3$

$= 8a + 2b - 3$

$f(-1) = 5 + b(-1) - 2(-1)^2 + a(-1)^3$

$= -a - b + 3$

Since, $f(2) = 2f(-1)$,

$8a + 2b - 3 = 2(-a - b + 3)$

$8a + 2b - 3 = -2a - 2b + 6$

$10a + 4b = 9$

Hence, it is proved.

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NUMERICAL CHALLENGE 4.8

Find the quadratic polynomial in x, which when divided by (x – 1), (x – 2) and (x – 3) leaves remainders of 11, 22 and 37 respectively.

Solution

Let the quadratic polynomial in x be $ax^2 + bx + c$

Now if $f(x) = ax^2 + bx + c$ is divided by (x – 1)

The remainder = $f(1) = a(1)^2 + b(1) + c = a + b + c$. It is given to be 11

$\therefore$ a + b + c = 11 .....(1)

When f(x) is divided by (x – 2)

remainder = $f(2) = a(2)^2 + b(2) + c = 4a + 2b + c$. It is given to be 22

$\therefore$ 4a + 2b + c = 22 .....(2)

When f(x) is divided by x – 3

remainder = $f(3) = a(3)^2 + b(3) + c = 9a + 3b + c$. It is given to be 37

$\therefore$ 9a + 3b + c = 37 .....(3)

Solving (1), (2) and (3) we get

a = 2, b = 5 and c = 4

$\therefore$ Required polynomial = $2x^2 + 5x + 4$

NUMERICAL CHALLENGE 4.9

If $f(x) = x^2 + 5x + p$ and $Q(x) = x^2 + 3x + q$ have a common factor then

(i) find the common factor (ii) show that $(p - q)^2 = 2(3p - 5q)$

Solution

Let the common factor be x – k

Remainder for the first polynomial $f(k) = k^2 + 5k + p = 0$

Remainder for the second polynomial $Q(k) = k^2 + 3k + q = 0$

$\therefore$ $k^2 + 5k + p = k^2 + 3k + q$

$\therefore$ $2k = q - p$

$\therefore$ $k = \frac{1}{2}(q - p)$

So, the common factor = $x - k = x - \frac{1}{2}(q - p) = x + \frac{1}{2}(p - q)$

Substituting $k = \frac{1}{2}(q - p)$ in $k^2 + 5k + p = 0$

We have $\left(\frac{q-p}{2}\right)^2 + 5\left(\frac{q-p}{2}\right) + p = 0$

$\therefore$ $\frac{(p-q)^2}{4} - \frac{5(p-q)}{2} + p = 0$

$\therefore$ $(p - q)^2 - 10(p - q) + 4p = 0$

$\therefore$ $(p - q)^2 = 10(p - q) - 4p = 6p - 10q = 2(3p - 5q)$

$\therefore$ $(p - q)^2 = 2(3p - 5q)$

We observe that k is a common root.

Hence, f(k) = 0, Q(k) = 0

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NUMERICAL CHALLENGE 4.10

The expression $2x^3 + ax^2 + bx - 2$ is divisible by $x + 2$ but leaves a remainder of 7 when divided by $2x - 3$. Determine

(i) the values of a and b, (ii) the remainder, when the expression is divided by $x - 2$.

Solution

Let $f(x) = 2x^3 + ax^2 + bx - 2$

(i) $f(-2) = 0 \Rightarrow 2(-2)^3 + a(-2)^2 + b(-2) - 2 = 0 \Rightarrow -16 + 4a - 2b - 2 = 0 \Rightarrow 4a - 2b = 18$

Dividing by 2, $2a - b = 9$ .....(1)

$$f\left(\frac{3}{2}\right)=7 \Rightarrow 2\left(\frac{3}{2}\right)^3 + a\left(\frac{3}{2}\right)^2 + b\left(\frac{3}{2}\right) - 2 = 7 \Rightarrow \frac{27}{4} + \frac{9}{4}a + \frac{3}{2}b - 2 = 7$$

$$\Rightarrow \frac{9}{4}a + \frac{3}{2}b = \frac{9}{4} \quad .....(2)$$

From (1), $b = 2a - 9$

Substituting in (2), we get $\frac{9}{4}a + \frac{3}{2}(2a - 9) = \frac{9}{4}$

$$\frac{9}{4}a + 3a - \frac{27}{2} = \frac{9}{4} \Rightarrow \frac{21}{4}a = \frac{63}{4} \Rightarrow a = 3 \text{ and}$$

$b = 2(3) - 9 = -3$

(ii) We have, $f(x) = 2x^3 + 3x^2 - 3x - 2$

$f(2) = 2(2)^3 + 3(2)^2 - 3(2) - 2 = 20$

$\therefore$ Remainder = 20

NUMERICAL CHALLENGE 4.11

Find the value of k for which $x^2 + 5kx + k^2 + 5$ is exactly divisible by $x + 2$ but not divisible by $x + 3$.

Solution

Let $f(x) = x^2 + 5kx + k^2 + 5$

as $f(-2) = 0$

$(-2)^2 + 5k(-2) + k^2 + 5 = 0$

$4 - 10k + k^2 + 5 = 0$

$k^2 - 10k + 9 = 0$

$(k - 1)(k - 9) = 0$

$\therefore$ $k = 1$ or $k = 9$

For $k = 1$, $f(x) = x^2 + 5(1)x + (1)^2 + 5 = x^2 + 5x + 1 + 5 = x^2 + 5x + 6$

When divided by $(x + 3)$,

$f(-3) = (-3)^2 + 5(-3) + 6 = 0$

Since for $k = 1$, $f(x)$ is divisible by $(x + 3)$, $k = 1$ is rejected.

$\therefore$ $k = 9$

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EXERCISE

Multiple choice questions

1. If the polynomial $x^{19} + x^{17} + x^{13} + x^{11} + x^7 + x^5 + x^3$ is divided by $(x^2 + 1)$, then the remainder is

(1) 1 (2) $x^2 + 4$ (3) $-x$ (4) x

2. If $(x - 2)$ is a common factor of $x^3 - 4x^2 + ax + b$ and $x^3 - ax^2 + bx + 8$, then the values of a and b are respectively

(1) 3 and 5 (2) 2 and –4 (3) 4 and 0 (4) 0 and 4

3. If the polynomial $x^6 + px^5 + qx^4 - x^2 - x - 3$ is divisible by $x^4 - 1$, then the value of $p^2 + q^2$ is

(1) 1 (2) 5 (3) 10 (4) 13

4. The expression $x^3 + gx^2 + hx + k$ is divisible by both x and $x - 2$ but leaves a remainder of 24 when divided by $x + 2$ then the values of g, h and k are

(1) $g = 10, h = -3, k = 0$ (2) $g = 3, h = -10, k = 0$

(3) $g = 10, h = -2, k = 3$ (4) None of these

5. The value of m if $2x^m + x^3 - 3x^2 - 26$ leaves a remainder of 226 when it is divided by $x - 2$.

(1) 0 (2) 7 (3) 10 (4) All of these

6. The expression $Ax^3 + x^2 + Bx + C$ leaves remainder of $\frac{21}{4}$ when divided by $1 - 2x$ and 18 when divided by x. Given also the expression has a factor of $(x - 2)$, the values of A, B and C are

(1) $A = 5, B = -9, C = 3$ (2) $A = 27, B = -18, C = 4$

(3) $A = 4, B = -27, C = 18$ (4) None of these

7. If $h(x) = 2x^3 + (6a^2 - 10)x^2 + (6a + 2)x - 14a - 2$ is exactly divisible by $x - 1$ but not by $x + 1$, then the value of a is

(1) 0 (2) –1 (3) 10 (4) 2

8. Given the polynomial is exactly divided by $x + 1$, and when it is divided by $3x - 1$, the remainder is 4. The polynomial gives a remainder $hx + k$ when divided by $3x^2 + 2x - 1$ then the values of h and k are

(1) $h = 2, k = 3$ (2) $h = 3, k = 3$

(3) $h = 3, k = 2$ (4) None of these

9. If $x^3 - hx^2 + kx - 9$ has a factor of $x^2 + 3$, then the values of h and k are

(1) $h = 3, k = 3$ (2) $h = 2, k = 2$

(3) $h = 2, k = 1$ (4) None of these

10. One of the factors of $x^3 + 3x^2 - x - 3$ is

(1) $x + 1$ (2) $x + 2$ (3) $x - 2$ (4) $x - 3$

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11. If $ax^2 + 2a^2x + b^3$ is divisible by $x + a$, then

(1) $a = b$ (2) $a + b = 0$

(3) $a^2 - ab + b^2 = 0$ (4) $a^2 + 2ab + b^2 = 0$

12. If $x^3 + 2x^2 + ax + b$ is exactly divisible by $(x + a)$ and $(x - 1)$, then

(1) $a = -2$ (2) $b = -1$ (3) $a = -1$ (4) $b = 1$

13. If $f(x) = ax^2 + bx + c$ is divided by $(bx + c)$, then the remainder is

(1) $\frac{c^2}{b^2}$ (2) $\frac{ac^2}{b^2} + 2c$ (3) $f\left(-\frac{c}{b}\right)$ (4) $\frac{ac^2 + 2b^2c}{b^2}$

14. $ax^4 + bx^3 + cx^2 + dx + e$ is exactly divisible by $x^2 - 1$, when

(1) $a + b + c + e = 0$ (2) $a + c + e = 0$ (3) $a + b = 0$ (4) $a + c + e = b + d = 1$

15. The remainder of $x^4 + x^3 - x^2 + 2x + 3$ when divided by $x - 3$ is

(1) 105 (2) 108 (3) 10 (4) None of these

16. If $x - 3$ is a factor of $x^3 + 3x^2 + 3x + p$, then the value of p is

(1) 0 (2) –63 (3) 10 (4) None of these

17. The value of $ax^2 + bx + c$ when $x = 0$ is 6. The remainder when dividing by $x + 1$ is 6. The remainder when dividing by $x + 2$ is 8. Then the sum of a, b and c is

(1) 8 (2) –1 (3) 10 (4) None of these

18. $x^n - y^n$ is divisible by $x + y$, when n is

(1) An odd positive integer (2) An even positive integer

(3) An integer (4) None of these

19. The remainder obtained when $5^{2009} + 13^{2009}$ is divided by 18 is

(1) 0 (2) –1 (3) 10 (4) None of these

20. The sum of the coefficients of the polynomial $p(x) = (3x - 2)^{17} (x + 1)^4$ is

(1) 16 (2) –1 (3) 10 (4) None of these

21. The remainder when x^{1999} is divided by $x^2 - 1$ is

(1) $-x$ (2) $3x$ (3) x (4) None of these

22. If $x + 1$ is a factor of $ax^4 + bx^3 + cx^2 + dx + e = 0$ then

(1) $a + c + e = b + d$ (2) $a + b = c + d$

(3) $a + b + c + d + e = 0$ (4) $a + c + b = d + e$

23. If $(x - 3)$ is the factor of $3x^3 - x^2 + px + q$ then

(1) $q + q = 72$ (2) $3p + q = 72$ (3) $3p + q = -72$ (4) $q - 3p = 72$

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24. For what values of n, (x + y) is a factor of $(x - y)^n$.

(1) For all values of n (2) 1

(3) Only for odd numbers (4) None of these

25. $f(x) = 3x^5 + 11x^4 + 90x^2 - 19x + 53$ is divided by x + 5 then the remainder is

(1) 100 (2) –100 (3) –102 (4) 102

26. If (x – 3), (x – 3) is a factors of $x^3 - 4x^2 - 3x + 18$; then the other factor is

(1) x + 2 (2) x + 3 (3) x – 2 (4) x + 6

27. If $f\left(\frac{-3}{4}\right)=0$; then for f(x), which of the following is a factor?

(1) 3x – 4 (2) 4x + 3 (3) –3x + 4 (4) 4x – 3

28. $f(x) = 16x^2 + 51x + 35$ then one of the factors of f(x) is

(1) x – 1 (2) x + 3 (3) x – 3 (4) x + 1

29. If $ax^3 + 9x^2 + 4x - 1$ is divided by (x + 2), the remainder is –6; then the value of 'a' is

(1) –3 (2) –2 (3) 0 (4) $\frac{33}{8}$

30. If $a^3 - 3a^2b + 3ab^2 - b^3$ is divided by (a – b), then the remainder is

(1) $a^2 - ab + b^2$ (2) $a^2 + ab + b^2$ (3) 1 (4) 0

31. If y = f(x) = mx + c; then f(y) in terms of x is

(1) mx + m + c (2) m + mc + c (3) $m^2x + mc + c$ (4) $m^2x + m^2c$

32. If 7 + 3x is a factor of $3x^3 + 7x$, then the remainder is

(1) $\frac{490}{9}$ (2) $\frac{-490}{9}$ (3) $\frac{470}{9}$ (4) None of these

33. The remainder when $f(x) = 3x^4 + 2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27}$ is divided by $g(x) = x + \frac{2}{3}$ is

(1) –1 (2) 1 (3) 0 (4) –2

34. The remainder when $1 + x + x^2 + x^3 + \ldots\ldots\ldots + x^{2006}$ is divided by x – 1 is

(1) 2005 (2) 2006 (3) 2007 (4) 2008

35. If (x + 1) is a factor of $x^4 + (p - 3)x^3 - (3p - 5)x^2 + (2p - 9)x + 6$ then the value of p is

(1) 1 (2) 2 (3) 3 (4) 4

36. If the remainder when the polynomial f(x) is divided by x – 1, x + 1 are 6, 8 respectively then the remainder when f(x) is divided by (x – 1)(x + 1) is

(1) 7 – x (2) 7 + x (3) 8 – x (4) 8 + x

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37. The remainder when x^{100} is divided by $x^2 - 3x + 2$ is

(1) $(2^{100} - 1)x + (-2^{100} + 2)$ (2) $(2^{100} + 1)x + (-2^{100} - 2)$

(3) $(2^{100} - 1)x + (-2^{100} - 2)$ (4) None of these

38. Find the remainder obtained when x^{2007} is divisible by $x^2 - 1$.

(1) x^2 (2) x (3) $x + 1$ (4) $-x$

39. The remainder on dividing $121^n - 25^n + 1900^n - (-4)^n$ by 2000 is

(1) 1 (2) 1000 (3) 100 (4) 0

40. The quotient and remainder when $x^{2002} - 2001$ is divided by x^{91} are respectively.

(1) $x^{91\times22}$, 2001 (2) x^{91}, 2001 (3) $x^{91\times21}$, –2001 (4) x^9, –2001

ALLEN

ANSWERS

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	3	3	3	2	2	3	4	2	1	1	1	3	3	2	2	2	1	2	1	1
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	3	1	3	4	3	1	2	4	4	4	3	2	3	3	4	1	1	2	4	3

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